Programme & Book of Abstracts

Tenth Scandinavian Logic Symposium (SLS 2018)
University of Gothenburg

11–13 June, 2018

Programme

Monday, 11th June

12:00–12:50 *Registration*

12:50–13:00 *Opening address*

13:00–14:00 *Invited talk:* **Joel David Hamkins**, *Set-theoretic potentialism and the universal finite set*

14:00–15:30 *Contributed talks:*

- 14:00 **Valentin Goranko and Antti Kuusisto**, *Logics for Propositional Determinacy and Independence*
- 14:30 **Fan Yang**, *Axiomatizing first-order consequences in inclusion logic*
- 15:00 **Gianluca Grilleti**, *Completeness for* `ClAnt` *and* `BWC` *fragments of* `inqBQ`

15:30–16:00 *Coffee break*

16:00–18:00 *Contributed talks:*

- 16:00 **Nemi Pelgrom**, *Inconsistency in informal mathematics*
- 16:30 **Michał Tomasz Godziszewski**, *Local disquotation and semantic (non)conservativeness*
- 17:00 **Bartosz Wcisło**, *Speed-up and Kripke–Feferman theory of truth*
- 17:30 **Cezary Cieśliński**, *On the conceptual strength of Weak and Strong Kleene evaluation schemata*

18:30–19:30 *Lindström lecture:* **Michael Rathjen**, *Progressions of theories and slow consistency*

19:30 *Welcome reception*

Tuesday, 12th June

10:00–11:00 *Invited talk:* **Luke Ong**, *Higher-order constrained Horn clauses and automatic program verification*

11:00–12:00 *Contributed talks:*

- 11:00 **David Ellerman**, *New logical foundations for information theory*
- 11:30 **Anupam Das**, *On the logical complexity of cyclic arithmetic*

12:00–13:30 *Lunch break*

13:30–15:30 *Contributed talks:*

- 13:30 **Bartosz Więckowski**, *Natural deduction with subatomic negation*
- 14:00 **Andreas Halkjær From, Helge Hatteland and Jørgen Villadsen**, *Teaching first-order logic with the natural deduction assistant (NaDeA)*

14:30 **Andreas Halkjær From**, *Formalized soundness and completeness of natural deduction for first-order logic*
15:00 **Sonia Marin**, *Proof theory for indexed nested sequents*

15:30–16:00 *Coffee break*

16:00–17:00 *Invited talk:* **Katrin Tent**, *Ampleness in strongly minimal structures*

17:00–18:00 *Contributed talks:*

17:00 **Paul Gorbow**, *Embeddings between non-standard models of set theory*
17:30 **Michał Tomasz Godziszewski**, Π^0_1-*computable quotient presentation of a nonstandard model of arithmetic*

19:00–22:00 *Conference dinner at Villa Belparc in Slottskogen*

Wednesday, 13th June

10:00–11:00 *Invited talk:* **Michael Rathjen**, *Bounds for the strength of the graph minor and the immersion theorem*

11:00–12:30 *Contributed talks:*

11:00 **Johan Lindberg**, *Point-free spaces of models*
11:30 **Mirko Engler**, *Relative interpretation and conceptual reduction of theories*
12:00 **Sebastian Eterovic**, *Categoricity of Shimura varieties*

12:30–14:00 *Lunch break*

14:00–16:00 *Contributed talks:*

14:00 **Olivier Bournez and Sabrina Ouazzani**, *Computing to the infinite with ordinary differential equations*
14:30 **Claes Strannegård**, *Artificial animals with dynamic ontologies*
15:00 **Dag Normann and Sam Sanders**, *On the mathematical and foundational significance of the uncountable*
15:30 **Torbjörn Lager**, *Rebranding Prolog*

16:00–16:30 *Coffee break & farewell*

16:30 *SLS business meeting*

Invited talks

Set-theoretic potentialism and the universal finite set

Mon, 13:00

Joel David Hamkins
CUNY

Providing a set-theoretic analogue of the universal algorithm, I shall define a certain finite set in set theory $\{x \mid \phi(x)\}$ and prove that it exhibits a universal extension property: it can be any desired particular finite set in the right set-theoretic universe and it can become successively any desired larger finite set in top-extensions of that universe. Specifically, ZFC proves the set is finite; the definition ϕ has complexity Σ_2 and therefore any instance of it $\phi(x)$ is locally verifiable inside any sufficiently large V_θ; the set is empty in any transitive model; and if ϕ defines the set y in some countable model M of ZFC and $y \subset z$ for some finite set z in M, then there is a top-extension of M to a model N of ZFC in which $\phi(x)$ defines the new set z. I shall draw out consequences of the universal finite set for set-theoretic potentialism and discuss several issues it raises in the philosophy of set theory. The talk will include joint work with W. Hugh Woodin, Øystein Linnebo and others. Questions and commentary concerning the talk can be made at: http://jdh.hamkins.org/set-theoretic-potentialism-sls-2018/

Higher-order constrained Horn clauses and automatic program verification

Tue, 10:00

Luke Ong
University of Oxford

We introduce constrained Horn clauses in higher-order logic, and study satisfiability and related decision problems motivated by the automatic verification of higher-order programs. Although satisfiable systems of higher-order clauses in the standard semantics do not generally have least models, by viewing these systems as a kind of monotone logic programs, we show that there are non-standard semantics that do satisfy the least model property. Moreover the respective satisfiability problems in the standard and non-standard semantics are inter-reducible. With a view to exploiting the remarkable efficiency of SMT solvers, we survey recent developments in the algorithmic solution of higher-order Horn systems by reduction to first order, and discuss related problems.

Mon, 18:00

Progressions of theories and slow consistency

Michael Rathjen

University of Leeds

The fact that “natural" theories, i.e. theories which have something like an “idea” to them, are almost always linearly ordered with regard to logical strength has been called one of the great mysteries of the foundation of mathematics. Using paradoxical methods, e.g. self-reference Rosser-style, one can distill theories with incomparable logical strengths and show that the degree structure of logical strengths is dense in that between two theories $S < T$ one can always find a third Q such that $S < Q < T$. But are there "natural" examples of such phenomena? We also know how to produce a stronger theory by adding the consistency of the theory. Can we get a stronger theory by adding something weaker than consistency that is still “natural”? These and other questions will be broached in the talk.

Wed, 10:00

Bounds for the strength of the graph minor and the immersion theorem

Michael Rathjen

University of Leeds

The graph minor theorem, GM, is arguably the most important theorem of graph theory. The strength of GM exceeds that of the standard classification systems of RM known as the “big five”. The plan is to survey the current knowledge about the strength of GM and other Kruskal-like principles, presenting lower and upper bounds

---

Tue, 16:00

Ampleness in strongly minimal structures

Katrin Tent

University Münster

The notion of ampleness captures essential properties of projective spaces over fields. It is natural to ask whether any sufficiently ample strongly minimal set arises from an algebraically closed field. In this talk I will explain the question and survey recent results on ample strongly minimal structures.

Contributed talks

Computing to the infinite with ordinary differential equations

Wed, 15:30

Olivier Bournez[a] and Sabrina Ouazzani[b]

[a]Ecole Polytechnique
[b]LACL, Université Paris-Est Créteil

In this talk, we present a class of differential equations that are equivalent to some transfinite time computation models and how they relate to each other.

We consider Continuous Ordinary Differential Equations (CODE). That is to say equations $x' = f(x)$, where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Such ordinary equations are known to always have solutions for a given initial condition $x(0) = x_0$, these solutions being possibly non unique.

We restrict our attention to the class of continuous functions such that for all x_0, the trajectory starting from x_0 is either locally constant, or with a specific property of local forward uniqueness. This class includes all common examples of functions, as well as all classical functions usually considered in mathematics as counterexamples related to unicity of solutions.

After having recalled the main results about Infinite Time Turing Machines (ITTM), we then prove the rather unexpected following result: CODE can be seen as models of computation over the ordinals (ITTM) and conversely in a very strong sense.

More specifically, this implies the next statements: To trajectories an ordinal can be associated, corresponding to some ordinal time of computation, and conversely.

From a technical angle, this is based on the proof of two (rather unexpected) facts that have never been established before, of independent interest: One can always accelerate computations even with an everywhere continuous dynamics. Continuous ordinary differential equations can be solved by ITTMs, even when non-unicity of trajectories (but only forward unicity) is assumed.

In a dual point of view, this yields facts for CODE, that were known for ITTM's, but not yet for continuous ordinary differential equations. This hence brings new perspectives on analysis in Mathematics.

Mon, 17:30

On the conceptual strength of Weak and Strong Kleene evaluation schemata

Cezary Cieśliński
University of Warsaw

One of the main research topics in the area of axiomatic theories of truth has been that of assessing their strength. A subtle measure of strength has been proposed by Fujimoto in [2]. Namely, denoting by L_T the result of adding a new predicate '$T(x)$' to the language of arithmetic, we say that the truth theory Th_1 is relatively truth-definable in Th_2 iff there is a formula $\theta(x) \in L_T$ such that for every $\psi \in L_T$, if $Th_1 \vdash \psi$, then $Th_2 \vdash \psi(\theta(x)/T(x))$.

If Th_2 defines the truth predicate of Th_1, then Th_2 is not *conceptually* weaker than Th_1, as Th_2 contains the resources permitting to reproduce the concept of truth of Th_1.

We will compare the conceptual strength of two axiomatic theories of truth: KF and WKF. The first one has been designed to capture Kripke's fixed-point construction based on Strong Kleene logic. The second one is based on the Weak Kleene evaluation schema.

In [2] Fujimoto proved that WKF is relatively truth-definable in KF. However, it has been an open question whether KF is relatively truth-definable in WKF.

We will provide the negative answer to this question. It should be emphasised that this is an absolute result, one that does not depend on the choice of language and coding. We consider this remarkable, because various important properties of Weak Kleene fixed-point construction are not absolute in this sense (see [1]).

[1] Cain, James and Damnjanovic, Zlatan, *On the Weak Kleene scheme in Kripke's theory of truth*, ***The Journal of Symbolic Logic***, vol. 56 (1991), no. 4, pp. 1452–1468.

[2] Fujimoto, Kentaro, *Relative truth definability of axiomatic truth theories*, ***Bulletin of Symbolic Logic***, vol. 16 (2010), no. 3, pp. 305–344.

Tue, 11:30

On the logical complexity of cyclic arithmetic

Anupam Das
University of Copenhagen

We study the logical complexity of proofs in cyclic arithmetic (CA), as introduced by Simpson in [1], in terms of quantifier alternations of formulae occurring. Writing $C\Sigma_n$ for (the logical consequences of) cyclic proofs containing only Σ_n formulae, our main result is that $I\Sigma_{n+1}$ and $C\Sigma_n$ prove the same Π_{n+1} theorems, for $n > 0$. Furthermore, due to the 'uniformity' of our method, we also show that CA and Peano Arithmetic (PA) proofs of the same theorem differ only elementarily in size.

The inclusion $I\Sigma_{n+1} \subseteq C\Sigma_n$ is obtained by proof theoretic techniques, relying on normal forms and structural manipulations of PA proofs. It improves upon the natural result that $I\Sigma_n \subseteq C\Sigma_n$. The converse inclusion, $C\Sigma_n \subseteq I\Sigma_{n+1}$, is obtained by calibrating the approach of [1] with recent results on the reverse mathematics of Büchi's theorem [3], and specialising to the case of cyclic proofs.

These results improve upon the bounds on proof complexity and logical complexity implicit in [1] and [2].

This abstract is based on the article [4]. The author is supported by a Marie Skłodowska-Curie fellowship, ERC project 753431.

[1] Alex Simpson, *Cyclic Arithmetic is Equivalent to Peano Arithmetic*, ***Proceedings of FoSSaCS '17***.

[2] Stefano Berardi and Makoto Tatsuta, *Equivalence of inductive definitions and cyclic proofs under arithmetic*, ***Proceedings of LICS'17***.

[3] Leszek Aleksander Kolodziejczyk and Henryk Michalewski and Pierre Pradic and Michał Skrzypczak, *The Logical Strength of Büchi's Decidability Theorem*, ***Proceedings of CSL'16***.

[4] Anupam Das, ***On the logical complexity of cyclic arithmetic***, Preprint: http://www.anupamdas.com/wp/log-comp-cyc-arith/.

New logical foundations for information theory

Tue, 11:00

David Ellerman

University of California Riverside

Quotient sets (= partitions = equivalence relations) are categorically dual to subsets, and there is now a logic of partitions ([1], [2]) dual to the usual Boolean logic of subsets. The quantitative version of the Boolean logic of subsets was finite logical probability theory, and the quantitative version of partition logic is the new logical theory of information ([3]) which shows what information is at the logical level (i.e., distinctions or 'dits'). The logical entropy of a partition is the probability that two independent drawings (with replacement) will yield a distinction (a pair of elements in distinct blocks). All the Shannon notions of simple, joint, conditional, and mutual entropy can be derived by a uniform requantifying transformation from the corresponding definitions of logical entropy–the latter being a measure in the sense of measure theory unlike the Shannon notions. Thus the logical theory of information displaces the Shannon theory as a foundational theory and repositions it as the specialized theory for coding and communications where the Shannon theory has been very successful. Logical information theory generalizes directly to QM giving a new approach to quantum information theory based on the corresponding notion of 'qudits' distinguishing quantum states. The fundamental theorem is that when projective measurement transforms the state's density matrix from a pure state to a mixed state (by the Lüders mixture operation), the sum of the absolute squares of the off-diagonal elements ("coherences") that are zeroed ("decohered") in the transformation is the quantum logical entropy generated by the measurement. Alternatively stated, the Hilbert-Schmidt distance between the pure state and the Lüders mixture state is the difference in their quantum logical entropies–which moreover is the probability that two independent measurements of the same state by the same observable give different eigenvalues.

[1] Ellerman, David. 2010. *The Logic of Partitions: Introduction to the Dual of the Logic of Subsets.* ***Review of Symbolic Logic*** 3 (2): 287–350.
[2] —. 2014. *An Introduction to Partition Logic.* ***Logic Journal of the IGPL*** 22 (1): 94–125.
[3] —. 2017. *Logical Information Theory: New Foundations for Information Theory.* ***Logic Journal of the IGPL*** 25 (5): 806–35.

Wed, 11:30

Relative interpretation and conceptual reduction of theories

Mirko Engler
Humboldt-University Berlin

Relative interpretations introduced by [1] were considered by many authors (e.g. [3]) to constitute conceptual reductions of theories. I will establish two versions of this claim by formalizing that everything which is expressed by a sentence φ in a theory S is also expressed by its relative translation $f(\varphi)$ in a theory T whenever S is relative interpretable in T via f ($S \prec_f T$) for at least one f (v.1) or for all such f (v.2). For this purpose, we axiomatize a relation $E(\varphi, T, \Phi)$ which captures that φ expresses in T that Φ. By proving a generalized version of Feferman's theorem of [2] that $PA + \neg Con_{pa} \prec PA$, we will finally disprove both v.1 and v.2.

Nevertheless, for some theories S and T we show that v.1 and v.2 actually holds. So it is natural to ask for a strengthening of relative interpretation for which v.1 and v.2 hold for all S and T. Taking into account [4], we prove that no strengthening of relative interpretation can validate v.2, while for v.1 there are some promising candidates for a conceptual reduction of theories.

[1] A. Tarski, A. Mostowski, R.M. Robinson, ***Undecidable theories***, Studies in logic and the foundation of mathematics, North-Holland, 1953.
[2] S. Feferman, *Arithmetization of metamathematics in a general setting*, ***Fundamenta Mathematicae***, vol. 49 (1960), no. 1, pp. 35–92.
[3] S. Feferman, *What rests on what? The proof-theoretic analysis of mathematics*, ***Philosophy of mathematics*** (J. Czermak, editor), Hoelder-Pichler-Tempsky, Wien, 1993, pp. 147–171.
[4] A. Visser, *Categories of theories and interpretations*, ***Lecture Notes in Logic*** (Logic in Teheran), (A. Enayat, I. Kalantari, M. Moniri, editors), vol. 26, The Association for Symbolic Logic, 2006, pp. 284–341.

Categoricity of Shimura varieties

Wed, 12:00

Sebastian Eterovic

University of Oxford

We propose a two-sorted model-theoretic structure for Shimura varieties. Using the methods of quasiminimality first developed by B. Zilber and then expanded in [1], we give necessary and suficient conditions for the theory determined by a Shimura variety to be categorical. These conditions are expressed in terms of Galois representations, so our main result establishes an equivalence between a model-theoretic condition and an arithmo-geometric condition. This condition has links to a conjecture of Pink and the Mumford-Tate conjecture.

For our main result we build upon the methods of [2] where the authors studied Shimura curves and prove categoricity unconditionally. Using a slightly more expressive language, we show that most of their methods can be extended to all dimensions, and only the final part needs the arithmo-geometric conditions.

This work follows an important series of results regarding categoricity of certain algebraic varieties arising from arithmetic: elliptic curves (M. Bays), multiplicative groups of algebraically closed fields (M. Bays and B. Zilber), semi-abelian varieties (B. Zilber), abelian varieties (M. Gavrilovich), commutative groups of finite Morley rank (M. Bays, B. Hart and A. Pillay), [2] for Shimura curves; among others.

[1] M. Bays, B. Hart, T. Hyttinen, M. Kesälä J. Kirby, *Quasiminimal structures and excellence*, ***Bulletin of the London Mathematical Society***, vol 46 (2014), no. 1, pp. 155–163.

[2] C. Daw A. Harris, *Categoricity of Modular and Shimura Curves*, ***Journal of the Institute of Mathematics of Jussieu***, vol 16 (2017), no. 5, pp. 1075–1101.

Formalized soundness and completeness of natural deduction for first-order logic

Tue, 14:30

Andreas Halkjær From

DTU Compute, AlgoLoG, Technical University of Denmark

We present a soundness and completeness proof of a natural deduction calculus for first-order logic, formalized in the interactive proof assistant Isabelle/HOL [1].

Our formalization is based on previous work by Stefan Berghofer [2]. The proof formalized by Berghofer uses Hintikka sets and only considers completeness for closed formulas [3]. We build on this proof to cover formulas with free variables via the following steps. First we universally close the formula, obtaining a derivation of its closure. Since we consider entailment in general we turn any judgment premises into implications as part of this. Then we eliminate each added quantifier with a fresh constant using the universal elimination rule from the calculus. Thereafter we use our own admissible rule to substitute the original variables for the fresh constants. Finally we show that the premises can always be weakened and use this to turn the implications back into premises, obtaining a derivation of the original formula.

We eliminate the universal closure with fresh constants instead of the free variables directly because we represent variables with de Bruijn indices; this makes reasoning about a chain of substitutions for free variables tricky, as each new substitution adjusts the variables from the previous ones.

Furthermore, we have updated Berghofer's formalization to use Isabelle's declarative proof style Isar [4]. Our formalization is available online. https://bitbucket.org/isafol/isafol/src/master/FOL_Berghofer/

A further development of the calculus is used for teaching at DTU [5].

[1] Tobias Nipkow, Lawrence C. Paulson and Markus Wenzel, ***Isabelle/HOL — A Proof Assistant for Higher-Order Logic***, vol. 2283, Lecture Notes in Computer Science, Springer, 2002.

[2] Stefan Berghofer, *First-Order Logic According to Fitting*, ***Archive of Formal Proofs***, August 2007. http://isa-afp.org/entries/FOL-Fitting.html

[3] Melvin Fitting, ***First-Order Logic and Automated Theorem Proving, Second Edition***, Graduate Texts in Computer Science, Springer, 1996.

[4] Markus Wenzel, *Isar — A Generic Interpretative Approach to Readable Formal Proof Documents*, ***Theorem Proving in Higher Order Logics, 12th International Conference, TPHOLs'99, September, Proceedings*** (Nice, France), (Yves Bertot, Gilles Dowek, André Hirschowitz, Christine Paulin-Mohring and Laurent Théry, editors), vol. 1690, Lecture Notes in Computer Science, Springer, 1999, pp. 167–184.

[5] Jørgen Villadsen, Andreas Halkjær From and Anders Schlichtkrull, *Natural Deduction and the Isabelle Proof Assistant*, *Proceedings 6th International Workshop on* ***Theorem proving components for Educational software*** (Gothenburg, Sweden), (Pedro Quaresma and Walther Neuper, editors), vol. 267, Electronic Proceedings in Theoretical Computer Science, Open Publishing Association, 2018, pp. 140–155. http://eptcs.org/paper.cgi?ThEdu17.9

Tue, 14:00

Teaching first-order logic with the natural deduction assistant (NaDeA)

Andreas Halkjær From, Helge Hatteland, and Jørgen Villadsen

DTU Compute, AlgoLoG, Technical University of Denmark

The natural deduction proof system is a popular way of teaching logic. It is also important in the philosophy of logic and the foundations of mathematics, in particular for systems of intuitionistic logic and constructive type theory, and it is used in many proof assistants along with automatic proof methods like the tableaux procedure and the resolution calculus.

The natural deduction assistant (NaDeA) has been used for teaching first-order logic to hundreds of computer science bachelor students since 2015 [1, 2]. NaDeA runs in a standard browser and is open source software. Upon completion of a natural deduction proof the student obtains a formal proof in the interactive proof assistant Isabelle/HOL [3] of not only the correctness of the student's natural deduction proof but also of the validity of the formula with respect to the classical semantics of formulas in first-order logic.

Our formalization of the syntax, semantics and the inductive definition of the natural deduction proof system extends work by Stefan Berghofer [4] and Melvin Fitting [5] but with a much more detailed soundness proof that can be examined and tested by the students. The corresponding completeness proof is also available but it is of course quite demanding. We describe the main advantages and disadvantages of using an advanced e-learning tools like NaDeA for teaching logic. Furthermore we briefly survey related and future work.

NaDeA can be used with or without installing Isabelle and is available online. https://nadea.compute.dtu.dk/

[1] Jørgen Villadsen, Alexander Birch Jensen and Anders Schlichtkrull, *NaDeA: A Natural Deduction Assistant with a Formalization in Isabelle*, ***IFCoLog Journal of Logics and their Applications***, vol. 4 (2017), no. 1, pp. 55–82.

[2] Jørgen Villadsen, Andreas Halkjær From and Anders Schlichtkrull, *Natural Deduction and the Isabelle Proof Assistant*, *Proceedings 6th International Workshop on* ***Theorem proving components for Educational software*** (Gothenburg, Sweden), (Pedro Quaresma and Walther Neuper, editors), vol. 267, Electronic Proceedings in Theoretical Computer Science, Open Publishing Association, 2018, pp. 140–155. http://eptcs.org/paper.cgi?ThEdu17.9

[3] Tobias Nipkow, Lawrence C. Paulson and Markus Wenzel, ***Isabelle/HOL — A Proof Assistant for Higher-Order Logic***, vol. 2283, Lecture Notes in Computer Science, Springer, 2002.

[4] Stefan Berghofer, *First-Order Logic According to Fitting*, ***Archive of Formal Proofs***, August 2007. http://isa-afp.org/entries/FOL-Fitting.html

[5] Melvin Fitting, ***First-Order Logic and Automated Theorem Proving, Second Edition***, Graduate Texts in Computer Science, Springer, 1996.

Π^0_1-computable quotient presentation of a nonstandard model of arithmetic

Tue, 17:30

Michał Tomasz Godziszewski

University of Warsaw

A *computable quotient presentation* of a mathematical structure $\mathcal{A}$ consists of a computable structure on the natural numbers $\langle \mathbb{N}, \star, *, \dots \rangle$, meaning that the operations and relations of the structure are computable, and an equivalence relation E on $\mathbb{N}$, not necessarily computable but which is a congruence with respect to this structure, such that the quotient $\langle \mathbb{N}, \star, *, \dots \rangle / E$ is isomorphic to the given structure $\mathcal{A}$. Thus, one may consider computable quotient presentations of graphs, groups, orders, rings and so on, for any kind of mathematical structure. In a language with relations, it is also natural to relax the concept somewhat by considering the *computably enumerable* quotient presentations, which allow the pre-quotient relations to be merely computably enumerable, rather than insisting that they must be computable.

At the 2016 conference Mathematical Logic and its Applications at the Research Institute for Mathematical Sciences (RIMS) in Kyoto, Bakhadyr Khoussainov outlined a sweeping vision for the use of computable quotient presentations as a fruitful alternative approach to the subject of computable model theory. In his talk, he outlined a program of guiding questions and results in this emerging area. Part of this program concerns the investigation, for a fixed equivalence relation E or type of equivalence relation, which kind of computable quotient presentations are possible with respect to quotients modulo E.

Khoussainov had made two specific conjectures in Kyoto:

Conjecture (Khoussainov).

1. *No nonstandard model of arithmetic admits a computable quotient presentation by a computably enumerable equivalence relation on the natural numbers.*
2. *Some nonstandard model of arithmetic admits a computable quotient presentation by a co-c.e. equivalence relation.*

I will report on the proof of first conjecture and present in details:

1. refutations of several natural variations of the second conjecture - obtained in a joint work with J. D. Hamkins,
2. proof of the central case of the second conjecture - obtained in a joint work with T. Slaman and L. Harrington.

In addition, I consider and settle the natural analogues of the conjectures for models of set theory.

Mon, 16:30

Local disquotation and semantic (non)conservativeness

Michał Tomasz Godziszewski
University of Warsaw

We analyse the (non)conservativeness properties of the classical locally disquotational theory of typed arithmetic truth TB and investigate its model-theoretic strength w.r.t the class of recursively saturated models of arithmetic. We first strengthen and generalise Cieśliński-Engström theorem on semantic (model-theoretic) non-conservativeness of TB over PA to a new result stating that TB is not semantically conservative over any complete extension of PA, including the True Arithmetic TA (= Th(N)). Cieslinski's and Engstrom's proof was insufficient to justify the latter and our proof provides a new argument that can be useful in further investigations of properties of axiomatic theories of truth. Further, we transfer the characterization of models of TB over set theories, which has some philosophical implications in the debate on deflationism w.r.t. the concept of mathematical truth. In the second part of the talk we separate the class of models of arithmetic expandable to a model of TB from the class of recursively saturated models, providing a completely new and conceptually simple proof of a result due to Łełyk and Wcisło

and contributing to the research in the hierarchy of model-theoretic strength of axiomatic truth theories. The main philosophical meaning of the first result is that it also ultimately strengthens contradiction to the claim purported by Ketland that the phenomenon of conservativeness boils down to adding compositionality principles to a given theory of truth. However, the theorem additionally invites philosophical interpretation contributing to the debate on the conservativeness in the field of deflationary theories of truth — it namely provides a reductio argument against considering semantic conservativeness as an adequate criterion for a truth theory, since the assumption that semantic conservativeness is an adequate criterion for a deflationary theory of truth strongly excludes TB from the class of adequate theories as a too strong one. I conclude by extending the results to the case of set theory, i.e. to disquotational theories of truth over ZFC taken as a base theory.

Logics for Propositional Determinacy and Independence

Mon, 14:00

Valentin Goranko[a] and Antti Kuusisto[b]

[a]Stockholm University

[b]University of Bremen

This talk is based on the recent paper [4], where we introduce and study formal logics for reasoning about propositional determinacy and independence. These relate naturally with the philosophical concept of supervenience [6], [7], which can also be regarded as a generalisation of logical consequence.

Propositional Dependence Logic $\mathcal{D}$ [9], [10], and Propositional Independence Logic $\mathcal{I}$ [5], [8], are recently developed logical systems, based on team semantics, that provide a framework for such reasoning tasks.

We introduce two new logics $\mathcal{L}_{\mathrm{D}}$ and $\mathcal{L}_{\mathrm{I}}$, based on Kripke semantics, and propose them as alternatives for $\mathcal{D}$ and $\mathcal{I}$, respectively. We analyse and compare the relative expressive powers of these four logics and also discuss how they relate to the natural language use and meaning of the concepts of determinacy and independence. We argue that $\mathcal{L}_{\mathrm{D}}$ and $\mathcal{L}_{\mathrm{I}}$ naturally resolve a range of interpretational problems that arise in $\mathcal{D}$ and $\mathcal{I}$. We also obtain sound and complete axiomatizations for $\mathcal{L}_{\mathrm{D}}$ and $\mathcal{L}_{\mathrm{I}}$ and relate them with the recently studied inquisitive logics and their semantics [2], [3], [1].

[1] Ivano Ciardelli. *Dependency as question entailment.* In Jouko Väänänen Samson Abramsky, Juha Kontinen and Heribert Vollmer, editors, ***Dependence Logic: theory and applications***, pages 129–181. Springer International Publishing, Switzerland, 2016.

[2] Ivano Ciardelli and Floris Roelofsen. *Inquisitive logic.* ***Journal of Philosophical Logic***, 40(1):55–94, 2011.

[3] Ivano Ciardelli and Floris Roelofsen. *Inquisitive dynamic epistemic logic.* ***Synthese***, 192(6):1643–1687, 2015.

[4] Valentin Goranko and Antti Kuusisto. *Logics for propositional determinacy and independence.* ***The Review of Symbolic Logic***, 2018, to appear.

[5] Erich Grädel and Jouko Väänänen. *Dependence and independence.* ***Studia Logica***, 101(2):399–410, 2013.

[6] Lloyd Humberstone. *Some structural and logical aspects of the notion of supervenience.* ***Logique et Analyse***, 35:101–137, 1992.

[7] Lloyd Humberstone. *Functional dependencies, supervenience, and consequence relations.* ***Journal of Logic, Language and Information***, 2(4):309–336, 1993.

[8] Juha Kontinen, Julian-Steffen Müller, Henning Schnoor, and Heribert Vollmer. *Modal independence logic.* ***Advances in Modal Logic 10***, pages 353–372, 2014.

[9] Jouko Väänänen. *Modal dependence logic.* In K. R. Apt and R. van Rooij, editors, ***New Perspectives on Games and Interaction***, pages 237–254. Amsterdam University Press, 2008.

[10] Fan Yang and Jouko Väänänen. *Propositional logics of dependence.* ***Annals of Pure and Applied Logic***, 167(7):557–589, 2016.

Tue, 17:00

Embeddings between non-standard models of set theory

Paul Gorbow

University of Gothenburg

This talk reports on some of the work in the speaker's forthcoming Ph.D. thesis.

In the first part of the talk, we will look at (1) a refined version of Friedman's theorem on the existence of embeddings between countable non-standard models of Power Kripke-Platek set theory with a strengthened separation axiom, and (2) an analogue of a theorem of Gaifman to the effect that any countable model of ZFC, that expands to a model of GBC + a schema saying that the class of ordinals is weakly compact, can be elementarily end-extended to a model with many automorphisms whose sets of fixed points equal the original model.

In the second part of the talk, we will see how these two theorems combine into a powerful technical machinery, yielding several results about non-standard models of set theory involving such notions as self-embeddings, their sets of fixed points and strong rank-cuts.

Completeness for `ClAnt` and `BWC` fragments of `inqBQ`

Mon, 15:00

Gianluca Grilletti

ILLC, Universiteit van Amsterdam

First-order inquisitive logic `inqBQ` [1] extends classical first-order logic to represent questions and logical relations between them. `inqBQ` is part of the family of Team Semantics [2] and is defined semantically following an approach similar to Dependence Logic [3]: a formula is evaluated with respect to a set (team) of models instead of a single one, and new logical symbols are introduced to capture relations between the models considered. Interpreting teams as a (possibly partial) information state and formulas as sentences, the semantics successfully captures if an information is enough to imply a statement or to resolve a question.

A natural deduction system for `inqBQ` was proposed in [1, §4], but the question whether the system is complete remains open. In the same Section, completeness results were shown for two fragments of the logic—*mention-all* and *mention-some* fragments—using slight variations of the system.

I propose completeness proofs for two other fragments of `inqBQ`. The first is the fragment `ClAnt` (c*lassical antecedent*), which extends both the *mention-all* and the *mention-some* fragments. This result is obtained by showing that saturated theories of `ClAnt` are characterized by the classical formulas they contain. The second is the `BWC` (*bounded world cardinality*) fragment, a large fragment which satisfies a version of the finite-model property. This result is obtained building a canonical model for the axiomatization in [1, §4] and studying the structure of the points with bounded E-width—those that see only a finite number of endpoints.

[1] Ivano Ciardelli, ***Questions in logic***, PhD thesis, ILLC, University of Amsterdam, 2016.

[2] Wilfrid Hodges, *Compositional semantics for a language of imperfect information*, ***Logic Journal of IGPL***, vol. 5 (4), pp. 539–563.

[3] Jouko Väänänen, ***Dependence Logic: A New Approach to Independence Friendly Logic***, London Mathematical Society Student Texts 70, Cambridge University Press, 2007.

Rebranding Prolog

Wed, 14:00

Torbjörn Lager

University of Gothenburg

> *Rebranding* is a marketing strategy in which a new name, term, symbol, design, or combination thereof is created for an established brand with the intention of developing a new, differentiated identity in the minds of consumers, investors, competitors, and other stakeholders.
> — *Wikipedia*

While the paradigms of imperative, functional and object-oriented programming have a vigorous following, the paradigm of logic programming with its flagship Prolog has fallen far behind. Various people both inside and outside the Prolog community have at various occasions voiced their fears about the future of the language, noting that there are too many incompatible Prolog systems around, resulting in a fragmented community and an ISO standard that few systems conform to.

I suggest *rebranding* as a strategy for reviving Prolog and to this end I present a proposal for a dialect called *Web Prolog*. Here is how I describe the language:

> Imagine a dialect of *Prolog* with processes and mailboxes and send and receive – all the means necessary for powerful concurrent and distributed programming. Alternatively, think of it as a dialect of *Erlang* with logic variables, backtracking search and a built-in database of facts and rules – the means for logic programming, knowledge representation and reasoning. Also, think of it as a *web logic programming language.* This is what *Web Prolog* is all about.

To prove that these are not just empty words of marketing, I shall demonstrate a number of tutorial-style examples showing how the language works. I will introduce the notion of Prolog *actors* (the locus of computation and interaction in Web Prolog) and show how primitives inspired by the Erlang programming language [1] can be used to spawn actors and make them talk to each other. On top of actors we shall build *pengines*, programming abstractions that give us first-class Prolog top-levels, and on top of a pengine we shall build an abstraction in the form of a meta-predicate for making *non-deterministic remote procedure calls* (NDRPC). I will go on to argue that by means of actors, pengines and the NDRPC abstraction we can create what I think of as the *programmable Prolog Web*, an extension of the Web which I believe might serve as an interoperability layer allowing the many incompatible Prolog systems to talk to each other, and in this way help to defragment the Prolog community. In the future, I also plan to take the initiative for the creation of a standard for the Web Prolog language under the auspices of W3C.

The work presented is an attempt to greatly refine and extend work described in [2]. The web-based GUIs used for some of the demonstrations is called SWISH and is described in [3].

[1] Joe Armstrong, ***Programming Erlang: Software for a Concurrent World***, Pragmatic Bookshelf, 2013.

[2] Torbjörn Lager and Jan Wielemaker, *Pengines: Web Logic Programming Made Easy*, ***Theory and Practice of Logic Programming***, vol. 14 (2014), no. 4-5, pp. 539–552.

[3] Jan Wielemaker, Fabrizio Riguzzi, Bob Kowalski, Torbjörn Lager, Fariba Sadri and Miguel Calejo, *Using SWISH to realise interactive web-based tutorials for logic-based languages*, ***Theory and Practice of Logic Programming***, (under review).

Point-free spaces of models

Wed, 11:00

Johan Lindberg
Stockholm University

The Joyal-Tierney representation theorem for Grothendieck toposes states that every such topos can be represented as the category of so-called equivariant sheaves on a localic groupoid. Thus, from the perspective of geometric theories, the classifying topos of a geometric theory can be constructed both from the syntax of the theory and from a localic groupoid, the points of which correspond to models of the theory.

We describe an ongoing project of giving (and exploiting) a more explicitly logical proof of the Joyal-Tierney representation theorem. A key aspect of our approach is the use of complete Heyting Algebra (cHA) valued sets instead of sheaves. One reason for pursuing this line of research is to further develop the constructive model theory for geometric and first-order intuitionistic logic with respect to cHA-valued sets, and to clarify its connections with more general topos-theoretic machinery.

This is joint work with Henrik Forssell.

Proof theory for indexed nested sequents

Tue, 15:00

Sonia Marin
University of Copenhagen

Modal logics were originally defined in terms of axioms in a Hilbert system and later in terms of their semantics in relational structures. Structural proof theory for modal logics, however, was considered a difficult topic as traditional (Gentzen) sequent calculus did not provide fully satisfactory (i.e. analytic and modular) proof systems even for some common modal logics.

Nonetheless, the proof theory of modal logics has received more attention recently, and some extensions of traditional sequents were successfully proposed to handle modalities. For example, *nested sequents* [3, 6, 1] are an extension of ordinary sequents to the structure of a tree that has shown fruitful in providing proof systems for modal logics.

However, the tree structure restricts the expressivity of nested sequents; in particular, it seems that they cannot give deductive systems for logics obeying the *Scott-Lemmon axioms*, which correspond semantically to a "confluence" condition on the relational structure [4].

Fitting introduced *indexed nested sequents* [2], an extension of nested sequents which goes beyond the tree structure to provide a proof system for classical modal logic K extended with an arbitrary (finite) set of Scott-Lemmon axioms. In this abstract we present our proof-theoretical study of Fitting's system, in particular its adaptation to the intuitionistic case. This is part of some joint work with Lutz Straßburger that was presented at *Tableaux'17* [5].

[1] Kai Brünnler, *Deep Sequent Systems for Modal Logic*, ***Archive for Mathematical Logic***, vol. 48, no. 6, 2009.
[2] Melvin Fitting, *Cut-Free Proof Systems for Geach logics*, ***IfCoLog Journal of Logics and their Applications***, vol. 2, no. 2, 2015.
[3] Ryo Kashima, *Cut-free sequent calculi for some tense logics*, ***Studia Logica***, vol. 53, no. 1, 1994.
[4] Edward J. Lemmon and Dana S. Scott, ***An Introduction to Modal Logic***, Blackwell, 1977.
[5] Sonia Marin and Lutz Strassburger, *Proof Theory for Indexed Nested Sequents*, ***26th International Conference on Automated Reasoning with Analytic Tableaux and Related Methods***, 2017.
[6] Francesca Poggiolesi, ***The Method of Tree-Hypersequents for Modal Propositional Logic***, Trends in Logic, Springer, 2009.

Wed, 15:00

On the mathematical and foundational significance of the uncountable

Dag Normann[a] and Sam Sanders[b]
[a]University of Oslo
[b]Center for Advanced Studies, München

Large parts of mathematics are studied *indirectly* via countable approximations, also called *codes*. Perhaps the most prominent example is Friedman-Simpson *Reverse Mathematics* ([6, 5]), which takes place in *second-order arithmetic*, i.e. only countable objects are directly available. It is then a natural question if anything is lost by the restriction to the countable imposed by second-order arithmetic. We show that this restriction fundamentally distorts mathematics. To this end, consider the following theorems which involve uncountable objects and cannot be proved in any (higher-type version) of Π^1_k-comprehension, while the countable versions (if existent) are weak.

1. *Cousin's lemma*: an open cover of $[0, 1]$ has a finite sub-cover, i.e. Heine-Borel compactness for (certain) uncountable covers.
2. *Lindelöf's lemma*: an open cover of $\mathbb{R}$ has a countable sub-cover.
3. *Besicovitsch and Vitali covering lemmas* as in [1, §2].
4. Basic properties (e.g. uniqueness) of the *gauge integral*; the latter is a generalisation of the Lebesgue and the improper Riemann integral, and provides a formalisation of Feynman's path integral.
5. *Neighbourhood Function Principle*; also provable in intuitionism.
6. The existence of *Lebesgue numbers* for uncountable covers.
7. The *Banach-Alaoglu theorem* for uncountable cover ([5, X.2.4]).
8. The *Heine-Young* and *Lusin-Young* theorems, the *tile theorem*, and the latter's generalisation due to Rademacher.

9. The *uniform* version of Heine's continuity theorem, historically established by Dini, Pincherle, Bolzano, and Lebesgue.
10. Many *uniform* theorems from the modern redevelopment of analysis based on Cousin's lemma (See e.g. [2, 6]).

These theorems are however typically provable in full (higher-order) second-order arithmetic and intuitionism, as well as often in (constructive) recursive mathematics. Moreover, the uniform version of Lindelöf's lemma for *Baire space* is acceptable in predicativist mathematics, but it nonetheless yields the impredicative system of Π^1_1-comprehension when combined with Feferman's μ operator. Finally, the exact strength of the Lindelöf lemma (provable in second-order arithmetic versus unprovable in ZF) is shown to crucially depend on the exact formulation. These results may be found in [3, 4].

[1] Pascal Aushcher and Lashi Bandara, ***Real Harmonic Analysis***, ANU Press, 2010.
[2] Robert Bartle and Donald Sherbert, ***Introduction to real analysis***, Wiley, 2000, pp. 404
[3] Dag Normann and Sam Sanders, *On the mathematical and foundational significance of the uncountable*, Submitted, 2017, arXiv: https://arxiv.org/abs/1711.08939.
[4] Dag Normann and Sam Sanders, *Uniformity in mathematics*, In preparation, 2018.
[5] Stephen Simpson, ***Subsystems of second-order arithmetic***, Cambridge University press, 2009.
[6] John Stillwell, ***Reverse mathematics, proofs from the inside out***, Princeton Univ. Press, 2018.

Inconsistency in informal mathematics

Mon, 16:00

Nemi Pelgrom
Stockholm University

It is central to mathematics that the systems we use are without inconsistencies. However, there are systems that are widely used which can be understood as only avoiding contradictions by a convention that forbids us to use the part of the system that would generate the contradiction. In this talk I will discuss the similarity between Russell's paradox and dividing by zero. Or in other words, I will try to show a significant theoretical difference between how Zermelo-Fraenkel set theory avoids Russell's paradox, and how division avoids dividing by zero. I will conclude that there is a interesting difference, and that the convention of avoiding their respective problematic instances by itself is not enough to remove any contradiction from a mathematical system. However, the convention is able to remove the undesirable consequences that inconsistent systems are able to produce. The findings in this paper presents us with examples of inconsistencies in informal mathematics, and a way to handle the problems of explosion, without having to reject the law of explosion.

Wed, 14:30

Artificial animals with dynamic ontologies

Claes Strannegård
Chalmers University of Technology

Despite the efforts made over the past half-century, systems that use formal logic to represent knowledge-bases and inference mechanisms for drawing conclusions from those knowledge-bases, have arguably not led to artificial intelligence. In fact, approaches based on logic alone face difficulties such as decision-making using incomplete knowledge, vulnerability to inconsistency, difficulty of formulating general learning principles, and difficulty of handling non-symbolic domains.

To realize the idea of artificial intelligence by imitating natural intelligence [?], one may need to zoom out and model a wider class of phenomena that includes evolutionary processes, nervous systems, reinforcement learning, and homeostatic decision-making [2]. This might in turn be done most naturally by extending the computational model beyond the logical framework.

A generic model of artificial animals that extends previous work [3] is presented. These artificial animals use reinforcement learning and have homeostasis as their only goal. They are equipped with sensors and motors in the form of propositional variables. Moreover, they use concepts and actions that are represented, respectively, as sequences of subsets of sensors and motors. Thus, concepts and actions can both be expressed as formulas of temporal logic. These artificial animals may start with arbitrary ontologies, i.e. sets of concepts and actions. Following rules for learning and forgetting these ontologies develop over time as concepts and actions are added and removed.

Examples are given of ecosystems where artificial animals develop and interact. Finally, results about the expressive and computational power of the generic model are presented.

[1] Wilson, Stewart W., *The Animat Path to AI*, ***Proceedings of the First International Conference on Simulation of Adaptive Behavior: From Animals to Animats*** (Paris, France), pp. 15–21, MIT Press, 1990.

[2] Yoshida, Naoto, *Homeostatic Agent for General Environment*, ***Journal of Artificial General Intelligence***, vol. 8, no. 1, 2017.

[3] Strannegård, Claes and Svangård, Nils and Lindström, David and Bach, Joscha and Steunebrink, Bas, *The Animat Path to Artificial General Intelligence*, ***Proceedings of the Workshop on Architectures for Generality and Autonomy at the International Joint Conference of Artificial Intelligence*** (Melbourne, Australia), 2017.

Speed-up and Kripke–Feferman theory of truth

Mon, 17:00

Bartosz Wcisło

University of Warsaw

Our talk concerns truth theories over Peano Arithmetic (PA). Truth theories are obtained by expanding the language of PA with a fresh unary predicate $T(x)$, a **truth predicate**, with the intended reading "x is a Gödel code of a true sentence", along with the axioms describing the behaviour of the newly added predicate.

One of the most classical truth theories has been introduced by Kripke and treated axiomatically by Feferman: its axioms state that the truth predicate obeys positive compositional clauses for all sentences, including the ones containing the truth predicate itself. Positive compositional clauses are equivalences such as:

$$\forall \phi, \psi \;\; T(\phi \wedge \psi) \equiv T\phi \wedge T\psi$$

but without a single axiom for the negated sentences. Instead, there are separate axioms for negations of atomic sentences, negations of conjunctions, double negations, and negations of quantified sentences treated via de Morgan laws.

A theory with such positive compositional clauses (but without induction for the formulae containing the truth predicate) is called KF^-. This theory allows to formalise a number of intuitive naive arguments involving the truth predicates and yet it is still consistent. Moreover, KF^- is **conservative** over PA, i.e., it does not prove any arithmetical theorems which are not provable in PA alone.

Recently, speed-up for conservative theories of truth began to be investigated. It has been shown by Ali Enayat, Matt Kaufmann, Mateusz Łełyk, and Albert Visser that the pure classical compositional theory of truth CT^- has at most polynomial speed-up over PA. (CT^- has all compositional axioms, including a single axiom for the negation, but the truth predicate does not apply to sentences containing that very predicate.) In other words, for any proof of an arithmetical sentence carried out in CT^-, one can find a proof of the same theorem in PA such that the proof using the truth predicate is at most polynomially shorter that the purely arithmetical one.

We extend these results and show that the theory KF^- also has at most polynomial speed-up over PA. Much like in the case of CT^-, our proof relies on the fact that a certain conservativeness proof for KF^- over fragments of arithmetic can be carried out in a uniform fashion.

Tue, 13:30

Natural deduction with subatomic negation

Bartosz Więckowski

Goethe-Universität Frankfurt am Main

In the language of first-order logic, negation is expressed by means of an operator which operates on formulae. This operator is very useful for the formalization of reasoning with sentential negation. However, since the narrowest scope it can take is an atomic formula, the negation operator of FOL is incapable of dealing with inferences whose validity depends on negations of predicate terms (e.g., negatively affixed gradable adjectives like 'unhappy'). We extend the language of FOL with subatomic negation operators and define an intuitionistic subatomic natural deduction system for the extension. Adapting the methods developed in [2], we obtain normalization and the subexpression property (a refinement of the subformula property) for the system. The first result allows us to formulate a proof-theoretic semantics for the subatomic operators. The system is in a position to adequately handle the interaction of the familiar superatomic operators of FOL with the newly added subatomic operators. In particular, it can give intuitively adequate formal expression to the interaction between predicate term negation of gradable/non-gradable adjectives (e.g., 'unhappy'/'non-prime') and formula negation. We compare our approach to predicate term negation with accounts which use contrary-forming formula operators (e.g., [1]) and accounts which make use of term logic (e.g., [3]). The research is a continuation of the naturalistic project of [4].

[1] McCall, S., *Contrariety*, ***Notre Dame Journal of Formal Logic***, vol. 8 (1967), no. 1/2, pp. 121–132.

[2] Prawitz, D., ***Natural Deduction: A Proof-Theoretical Study***, Stockholm: Almqvist & Wiksell, 1965. (Reprint: Mineola/NY, Dover Publications, 2006.)

[3] Sommers, F. and Englebretsen, G., ***An Invitation to Formal Reasoning: The Logic of Terms***, Aldershot: Ashgate, 2000.

[4] Więckowski, B., *Subatomic natural deduction for a naturalistic first-order language with non-primitive identity*, ***Journal of Logic, Language and Information***, vol. 25 (2016), no. 2, pp. 215–268.

Axiomatizing first-order consequences in inclusion logic

Mon, 14:30

Fan Yang
University of Helsinki

Inclusion logic [2] is a recent variant of *dependence logic*, which was introduced by Väänänen [8] as an extension of first-order logic with a new type of atomic formulas that specify explicitly the *dependence* between variables. Inclusion logic, instead, extends first-order logic with the so-called *inclusion atoms* that specify the *inclusion* relation between variables. Inclusion logic as well as dependence logic adopt the so-called *team semantics* of Hodges [6, 7], and evaluate formulas on *sets* of assignments (called *teams*), instead of single assignments as in the usual semantics.

Teams can be viewed as relations, which are second-order objects. Essentially for this reason, dependence logic and one of its major variant called independence logic [4] both have the same expressive power as existential second-order logic, and they are thus not (effectively) axiomatizable. Nevertheless, first-order consequences of these two logics are axiomatizable and explicit natural deduction systems are given in [1, 5]. In this talk, we present a similar result for inclusion logic, which is known to have the same expressive power as positive greatest fixed-point logic [3]. We argue that inclusion logic is not (effectively) axiomatizable in full by showing that *well-foundness* of orders is definable in the logic, and we introduce a natural deduction system that is complete in the sense that

$$\Gamma \vdash \alpha \iff \Gamma \models \alpha \tag{1}$$

holds whenever α is first-order. While our result is obtained by applying the methodology developed in [1, 5], the system we present for inclusion logic has more desirable properties in that it has more natural rules, and (via a trick of [9]) the completeness theorem (1) holds for arbitrary (possibly open) formulas (in contrast to the systems of [1, 5] for which (1) holds only when all of the formulas involved have to be sentences without free variables).

[1] J. Kontinen, and J. Väänänen, *Axiomatizing first-order consequences in dependence logic.* ***Annals of Pure and Applied Logic 164***, 11 (2013).

[2] P. Galliani, *Inclusion and exclusion dependencies in team semantics: On some logics of imperfect information.* ***Annals of Pure and Applied Logic***, 163(1):68 – 84, 2012.

[3] P. Galliani, and L. Hella, *Inclusion logic and fixed point logic.* In ***Computer Science Logic 2013***, Schloss Dagstuhl, pp. 281–295 (2013).

[4] E. Grädel and J. Väänänen, *Dependence and independence.* ***Studia Logica***, 101(2):399-410, 2013.

[5] M. Hannula, *Axiomatizing first-order consequences in independence logic.* ***Annals of Pure and Applied Logic 166***, 1 (2015), 61–91.

[6] W. Hodges, *Compositional Semantics for a Language of Imperfect Information*, ***Logic Journal of the IGPL***, vol. 5, pp. 539-563, 1997.

[7] W. Hodges, *Some Strange Quantifiers*, ***Structures in Logic and Computer Science: A Selection of Essays in Honor of A. Ehrenfeucht***, Springer, pp. 51-65, 1997.

[8] J. Väänänen, ***Dependence Logic: A New Approach to Independence Friendly Logic***, Cambridge University Press, 2007.
[9] F. Yang, *Negation and Partial Axiomatizations of Dependence and Independence Logic Revisited*, ***WoLLIC 2016***, Springer-Verlag, 2016, pp. 410-431